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Preliminary Design of Composite Wings for Buckling, Strength, and Displacement Constraints

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An unstiffened panel buckling constraint for balanced, symmetric, laminated composites is included on the global design level in a mathematical programming structural optimization procedure for designing wing structures. Constraints are introduced by penalty functions, and Newton's method based on approximate second derivatives of the penalty terms is used as the search algorithm to obtain minimum-mass designs. Constraint approximations used during the optimization process contribute to the computational efficiency of the procedure. For some constraints, such as buckling, the use of constraint approximations may result in significant errors. By using a conservative constraint approximation, the effect of these errors can be minimized. A criterion is developed that identifies the appropriate conservative form of the constraint approximations that are used with the optimization procedure. Minimum-mass design results are obtained for a multispar high aspect ratio wing subjected to material strength, minimum-gage, displacement, panel buckling, and twist constraints. The material systems considered for the examples are all graphite-epoxy, graphite-epoxy with boron-epoxy spar caps, and all aluminum. The composite material designs are shown to have an advantage over the aluminum designs since they can often satisfy additional constraints with only small mass increases. This advantage results from the composite material's additional design freedom of changing relative percentages of different lamina orientations rather than just total laminate thickness.

Introduction

ADVANCED composite materials offer an attractive potential for reducing the structural mass of modern wing designs. To achieve this potential, minimum-mass designs must be provided that simultaneously satisfy a multitude of local and global wing design constraints, such as material strength, minimum-gage, buckling, displacement, and flutter constraints. Designing a structural component of laminated construction is inherently more involved than designing its metal counterpart because of the additional design freedom of changing the relative percentages of the different layers or laminae rather than just the total laminate thickness. The development of preliminary design tools based on structural optimization procedures is expected to offer the designer a cost-effective capability to generate minimum-mass structural designs, and the tools are expected to have enough generality to use the directional dependence of the material laminae to best advantage.

Although other techniques may be highly suitable for structural optimization subject to any single constraint, mathematical programming methods offer the generality required to deal with a multiconstraint design environment. Some constraints, such as flutter and displacement, affect only the general or global design of a wing structure, while other constraints, such as panel buckling and local buckling of panel elements, control local dimensions. Changing dimensions or design variables to satisfy any of these constraints can change the internal load distribution in a wing. One approach[1] to account for internal load redistribution is to couple the local and global design effects by an iterative or multilevel method that alternates between the local and global levels of design. Such an approach is very useful when the local design problem is complex in itself, as in the case of a stiffened panel where many local parameters are needed to define each wing panel. Another approach to account for internal load redistribution is to have changes in local design variables immediately reflected in changes in the internal loads by dealing with local constraints directly on the global level. This can be done readily for simpler local design problems, such as unstiffened or sandwich panels, by making the local design variables into global ones and optimizing the entire wing design problem on the global level. The advantage of this global approach is that it is more likely to result in a true minimum-mass design. Furthermore, when applied to such simpler cases, the global design approach could be used to verify the reliability of any particular multilevel design approach.

The present paper extends an efficient mathematical programming optimization procedure,[2] and includes wing twist and panel buckling constraints with earlier developed material strength, displacement, and minimum-gage constraints. In this procedure, the panel buckling constraint for laminated composite panels is included on the global design level for the first time. Derivatives of the constraints with respect to the design variables are determined analytically. These derivatives, used to determine search directions and to obtain approximate expressions for the constraint functions, are an important factor in the efficiency of the optimization procedure. Two forms of constraint approximations are examined in this study, and a criterion is developed to

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etermine which of the two forms is the more conservative for given set of constraint derivatives. The form of the conraint approximation is important, since it can have a gnificant effect on the computational efficiency of the :arch procedure.

In this paper, the design procedure is used to provide :veral examples of aluminum and composite multispar high spect ratio wing designs subject to a number of diverse esign constraints. The effects of various constraints on the iinimum-mass designs of these wings are evaluated and the dvantages of the composite designs over the aluminum esigns are assessed.

Design Method

ptimization Procedure

The design problem is to find a vector of design variables v 'ith components v_j; $j=1, \ldots, N$, that minimizes the mass $m(v)$ of a wing structure subject to the constraint functions

$$g_i(v) \geq 0 \qquad i=1, \ldots, k \tag{1}$$

or example, a typical form of stress constraint is:

$$g = 1 - \frac{\sigma}{\sigma_{\max}} \geq 0 \tag{2}$$

/here $\sigma_{\max}$ is the allowable stress. The constrained opimization problem is transformed into a series of unonstrained minimization problems and solved by the equence of unconstrained minimizations technique (SUMT)[3] sing the quadratic extended interior penalty function defined 1 Ref. 2. The resulting transformed problem is to find the ninimum of a function $P(r)$ for a sequence of decreasing alues of r, where

$$P(r) = m(v) + r \sum_{i=1}^{k} f_i(v) \tag{3}$$

nd the function $f_i(v)$ associated with the ith constraint is efined as:

$$f_i(v) = \begin{cases} 1/g_i & \text{if } g_i \geq g_0 \\ 1/g_0[(g_i/g_0)^2 - 3(g_i/g_0) + 3] & \text{if } g_i \leq g_0 \end{cases} \tag{4}$$

'he function $f_i(v)$ is thus defined as an interior penalty unction in most of the feasible design domain ($g_i \geq g_0$). It is lefined as a quadratic exterior penalty function in a small part of the feasible domain ($g_i \leq g_0$) and in the infeasible domain, hus permitting incursions into the infeasible domain. The ransition parameter g_0 used in Eq. (4) varies with r as lescribed in Ref. 2. For each value of the constraint weighting actor r, an unconstrained minimization of $P(r)$ is carried out ısing Newton's method with approximate second derivatives of the penalty terms.[4] As r becomes small, the design 'ariables v obtained by minimizing $P(r)$ approach the desired ninimum-mass design. By using the formulation of Refs. 2 .nd 4 the number of analyses needed by the optimization rocedure is small and independent of the number of design ariables. The generality of this optimization procedure llows additional constraints, such as panel buckling contraints, to be imposed on the global design process of a wing y including appropriate constraint functions in the sumnation in Eq. (3).

3uckling Constraints

The panel buckling constraints used by the optimization procedure are represented by an interaction equation based on in engineering approximation that relates uniaxial buckling tresses and pure shear buckling stresses for simply-supported laminated plates. This constraint function for buckling is:

$$g = 1 - \left(\frac{N_x}{N_{x_{cr}}}\right) - \left(\frac{N_y}{N_{y_{cr}}}\right) - \left(\frac{N_{xy}}{N_{xy_{cr}}}\right)^2 \tag{5}$$

where N_x, N_y, and N_{xy} are normal and shear stress resultants and $N_{x_{cr}}$, $N_{y_{cr}}$, and $N_{xy_{cr}}$ are corresponding buckling stress resultants. The uniaxial buckling stress resultant for a rectangular orthotropic plate is:[5]

$$N_{x_{cr}} = -\pi^2 [D_{11}(i/a)^2 + 2(D_{12} + 2D_{66})(k/b)^2 + D_{22}(k/b)^4 (a/i)^2] \tag{6}$$

where a and b are the panel side lengths, i and k are wave numbers that are varied to minimize the buckling stress resultant; and D_{11}, D_{22}, D_{12}, and D_{66} are bending stiffnesses. A similar expression is used for $N_{y_{cr}}$. The critical shear stress resultant $N_{xy_{cr}}$ of an infinite orthotropic strip of width b depends on a parameter[5]

$$K = [D_{11}D_{22}/(D_{12} + 2D_{66})^2]^{1/2} \tag{7}$$

and is given by

$$N_{xy_{cr}} = \begin{cases} 4B/b^2 (D_{11}D_{22}^3)^{1/4} & \text{if } 1 \leq K \leq \infty \\ 4B/b^2 [D_{22}(D_{12} + 2D_{66})]^{1/2} & \text{if } 0 \leq K \leq 1 \end{cases} \tag{8}$$

The factor B in Eq. (8) is determined by linear interpolation from Table 4-3 of Ref. 5. The bending stiffnesses for laminated plates are found by summing the contributions of n laminae of a composite laminate, and an example expressed in the notation of Ref. 6 is given by

$$D_{11} = \frac{1}{3} \sum_{k=1}^{n} (\bar{Q}_{11})_k (h_k^3 - h_{k-1}^3) \tag{9}$$

where $\bar{Q}_{11}$ is a lamina material property and h is the distance from the kth lamina to the laminate reference surface.

Other Constraints and Constraint Derivatives

The other design constraints considered herein are material strength, minimum-gage, displacement, and twist constraints. The optimization procedure requires the derivatives of each constraint with respect to the design variables. These derivatives can be computed analytically and, consequently, the large number of analyses associated with finite-difference derivatives is avoided. The derivatives of displacement and stress constraints can be computed analytically in a straightforward manner as shown in Ref. 2. The derivatives of strain and twist constraints are readily obtainable from the displacement derivatives. The derivatives of the buckling constraints are obtained by differentiating Eq. (5) which requires the analytical derivatives of expressions such as Eqs. (6, 8, and 9).

Constraint Approximations

It is now common practice (e.g., Ref. 7) to express the constraints as approximate, explicit functions of the design variables. By making such constraint approximations, the number of analyses required to find a minimum-mass design is reduced considerably. Instead of performing a new analysis with each small change in the design variables, it is only necessary to update the approximations periodically during the optimization process. The simplest form of constraint approximation is a linear function of the design variables. Another form of approximation that is often used (e.g., Refs. 2, 7, and 8) is a linear function of the inverse of the design variables. This approximation is exact for the stresses and displacements of a statically determinate structure. Several examples presented in Ref. 9 show that the inverse approximation is more accurate for stresses and displacements than the linear approximation for a statically indeterminate structure. In performing the calculations for the examples reported herein, it was found that both forms of approximations may result in much larger errors for the buckling loads than for the stresses and displacements. Consequently, it is more important to use a conservative approximation for buckling constraints than for stress and displacement constraints.

To determine which is the more conservative, the two forms of approximation are compared by considering a constraint function $g(v)$ which, together with its first derivatives, is known for some reference design-variable vector v_0. Then, by using the first two terms in a Taylor series, g may be approximated for another v (with components v_i; $i = 1, \ldots, N$) by either an expression g_L that is linear in the design variables

$$g_L(v) = g(v_0) + \sum_{i=1}^{N} (v_i - v_{oi}) \frac{\partial g(v_0)}{\partial v_i} \tag{10}$$

or an expression g_I that is linear in the inverse of the design variables

$$g_I(v) = g(v_0) + \sum_{i=1}^{N} \frac{v_{oi}}{v_i} (v_i - v_{oi}) \frac{\partial g(v_0)}{\partial v_i} \tag{11}$$

Since the constraint equations used herein are expressed as $g(v) \geq 0$ [see Eq. (1)], g_I is more conservative than g_L when $g_I < g_L$ and vice versa. A criterion for determining the more conservative approximation is obtained by subtracting Eq. (10) from Eq. (11) to get

$$g_I(v) - g_L(v) = -\sum_{i=1}^{N} \frac{1}{v_i} (v_i - v_{oi})^2 \frac{\partial g(v_0)}{\partial v_i} \tag{12}$$

and noting that v_i are positive for physical design variables such as thicknesses and areas. Therefore, from Eq. (12), g_I is more conservative than g_L when

$$\partial g(v_0) / \partial v_i > 0 \tag{13}$$

A typical form of a stress constraint is given by Eq. (2). Then, Eq. (13) implies that the inverse approximation is more conservative when $\partial \sigma / \partial v_i < 0$. For most structural problems this criterion is easily met because increasing thicknesses or areas usually results in lowering the absolute values of stresses and displacements. However, for indeterminate or redundant structures, increasing the size of a member can increase the stress in another member of the structure. An example of such behavior is demonstrated by the simple truss shown in Fig. 1. If all members are made of the same material, the stress σ_2 in the smaller part of the middle bar is:

$$\sigma_2 = \frac{4A_1 F}{\sqrt{2} A_2 (A_1 + A_2) + 4A_1 A_2} \tag{14}$$

Since the derivative of σ_2 with respect to A_1 is always positive, increasing A_1 increases σ_2.

In the optimization procedure used herein a hybrid approximation $g_H(v)$ is employed that always uses the more

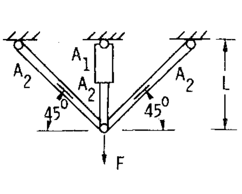

Fig. 1 Simple truss example.

conservative of the two approximations considered. Th hybrid approximation is given by:

$$g_H(v) = g(v_0) + \sum_{i=1}^{N} B_i \frac{\partial g(v_0)}{\partial v_i} \tag{15}$$

where

$$B_i = \begin{cases} (v_i - v_{oi}) & \text{if } \dfrac{\partial g(v_0)}{\partial v_i} < 0 \\[2ex] \dfrac{v_{oi}}{v_i}(v_i - v_{oi}) & \text{if } \dfrac{\partial g(v_0)}{\partial v_i} > 0 \end{cases} \tag{16}$$

With this hybrid approximation, a given constraint may hav a linear approximation with respect to one design variable an an inverse approximation with respect to another desig variable.

It was found in this study that buckling loads are ver sensitive to changes in the values of the design variables Therefore, while noncritical strength and displacemen constraints were deleted during portions of the design searcl procedure, all buckling constraints were retained at all times.

Computer Program

The computer program used for performing th calculations for the examples given in this paper is modification of the WIDOWAC program.[10] In the program a wing structure is modeled by a finite-element representatio that includes rod elements for rib and spar caps, constan strain triangular membrane elements for cover panels, an shear web elements for ribs and spars. Quadrilateral element made up of the average of four triangles are typically used fo modeling cover panels. The program is limited to symmetri airfoil sections. Design variables define thicknesses or area of structural elements.

For the example problems described in the following, a initial value of 0.3 was used for the transition parameter g_0 i Eq. (4). The initial value of r was chosen so that any constraint that is exactly critical (i.e., $g = 0$) contributes a penalt term that is equal to one-third of the original mass. After eac unconstrained minimization, r was reduced by a factor of 2 and the optimization procedure was terminated when th penalty function was reduced to less than 1% of the mass.

Description of Wing Models

In this study, a swept, high aspect ratio structural wing box typical of a subsonic transport aircraft wing in the 73,000 kg gross takeoff mass class is modeled using a multispar construction concept. The wing has a span of approximately 33 m and an aspect ratio of about 7.7. The depth of the wing box is assumed to taper from 48.8 cm at the root to 14.2 cm at the wing tip. Since the computer program used in the study was developed for symmetric airfoils, only the upper half of the wing needs to be modeled. A 4 internal spar configuration of the wing is shown in Fig. 2. Other configurations having 3, 5, and 6 internal spars were also studied.[11] Three material systems are used in the study—all graphite-epoxy, graphite-epoxy with boron-epoxy spar caps, or all 7075 aluminum alloy. The material properties used in this study are given in Table 1. For the aluminum models, a single membrane element is used to represent a panel bounded by ribs and spars. For the composite material models four membrane elements, one for each of four material orientations, are stacked to form a laminate representing a panel bounded by ribs and spars. A balanced, symmetric laminate is generated internally in the computer program to represent the stacking sequence $[-45°_i/+45°_i/0°_j/90°_k/0°_j/+45°_i/45°_i)$ for the composite panels. Here i,j, and k are not integer numbers

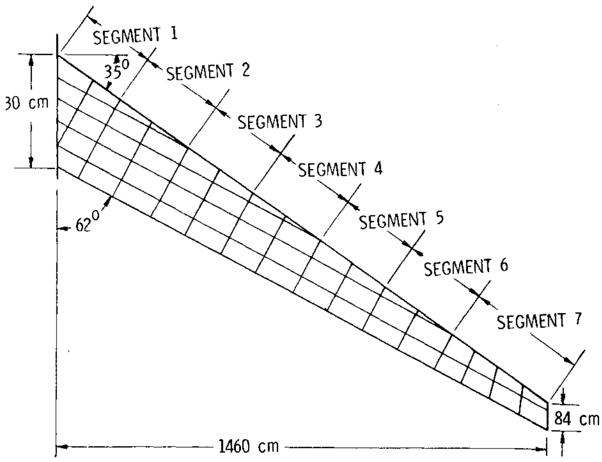

Fig. 2 Wing model.

ſ laminae, but are used to indicate that the thickness of each mina is determined by the program and that the +45 deg ıd −45 deg material are controlled by one design variable ıd the 0 and 90 deg materials by two independent design ıriables. Each rib and spar element in the composite material ıodels is represented by one quasi-isotropic graphite-epoxy ıear web, and the spar caps are represented by either 0 deg ·aphite-epoxy or 0 deg boron-epoxy rod elements.

The finite-element models have 73 grid points and 201 egrees of freedom. The aluminum model has 248 finite ements and the composite models have 413. The wing is ivided into seven spanwise segments (see Fig. 2) for design ıriable distribution purposes. The basic design variable istribution has one design variable assigned to the thickness f each material orientation of the skin in each segment, one esign variable assigned to the thickness of each spar web, nd one design variable assigned to the area of each spar cap 'or a total of 33 design variables for the composite models nd 19 for the aluminum model). This basic design variable istribution imposes constant chordwise cover thicknesses ithin each wing segment and spar webs and caps that are ɔnstant along each spar. No design variables are assigned to ıe ribs, which are held at a constant thickness of 0.34 cm ith a small number of ribs near the wing root being 0.67 cm. . refined design variable distribution is also used to assess the 'fect of chordwise cover thickness variation and spanwise ɔar web and cap variation. Between two and four design variables are assigned to the thickness of each material orientation of the skin in each wing segment (depending on the segment), and up to four design variables are assigned to the shear webs and caps of each spar. A total of 57 design variables are used with this refined design variable distribution for the aluminum model, and 99 design variables are used with the composite models.

Pressure and inertia loads corresponding to a 3.75 g ultimate design condition with a 72,574 kg gross mass are imposed on the wing box with no load redistribution due to aeroelastic effects. The wing is considered clamped at the root. The design constraints applied to the wing models are material strength, minimum-gage, displacement, twist, and panel buckling constraints and are summarized in Table 1.

Results and Discussion

One objective of the design study is to better understand the differences between composite material designs and metal designs. The other objectives are to assess the effectiveness of using boron-epoxy spar caps in an otherwise graphite-epoxy design, and to compare a wing panel designed on the global level with a single panel designed independent of the rest of the wing.

Effect of Various Design Constraints

The individual and cumulative effects of the various design constraints on the composite and aluminum models are studied to help understand the differences between the resulting designs for the basic design variable distribution described earlier. The design constraints imposed on the models considered consist of various combinations of material strength, minimum-gage, displacement, panel buckling, and twist constraints. The final wing mass and wing tip displacement results for the various constraint combinations are presented in Table 2. The cover skin thickness distribution results for the all-graphite-epoxy model, subject to the various constraint combinations, are shown in Fig. 3 for the basic design variable distribution.

The stiffness-to-mass advantage of composite materials over aluminum is reflected in Table 2. For the designs subjected only to strength and minimum-gage constraints, for example, the wing tip deflection of the heavier aluminum design is about twice that of the composite designs. Adding a constraint on wing tip displacement causes a very large mass increase for the aluminum design compared to the composite designs. The composite designs consist mainly of 0 deg material with the 90 and ±45 deg material sized at minimum

Table 1 Material properties and design allowables

	Aluminum	Graphite-epoxy	Boron-epoxy
Longitudinal Young's modulus, GPa	72.3	130.9	206.7
Transverse Young's modulus, GPa	72.3	13.0	18.6
Shear modulus, GPa	27.6	6.4	6.4
Major Poisson's ratio	0.33	0.38	0.21
Density, kg/m^3	2770	1610	2008
Stress allowables, MPa			
Normal	503	—	—
Shear	290	—	—
Strain allowables			
Extensional	—	0.004	0.004
Shear	—	0.015	0.015
Minimum gages			
Skin thickness, mm	0.51	0.28[a]	0.28[a]
Web thickness, mm	0.51	0.56	0.56
Cap areas, cm^2	0.65	0.65	0.65
Wing tip displacement, cm	165	165	165
Twist, deg			
Wing tip	3	3	3
63% of semispan	0.1	0.1	0.1

[a] Thickness per lamina.

Table 2 Basic configuration final design results

Material system	Constraints[a] S+M	S+M+D	S+M+B	S+M+D+B	S+M+D+B+T
Aluminum					
Mass, kg	1068	2528	2603	2623	3177
Displacement, cm	502	165	166	164	134
Graphite-epoxy					
Mass, kg	747	996	1612	1624	1722
Displacement, cm	271	165	225	164	120
Graphite-epoxy with boron-epoxy spar caps					
Mass, kg	703	879	1533	1623	1676
Displacement, cm	259	165	219	164	135

[a] S = strength, M = minimum gage, D = displacement, B = buckling, and T = twist.

gage, as shown in Fig. 3, by the constraint cases labeled S + M and S + M + D.

Adding panel buckling constraints to the material strength and minimum-gage constraints results in a large increase in mass for both the composite and aluminum models. This increase occurs because most of the wing panels are buckling critical rather than strength critical. (The effect of the buckling constraints on the mass is more pronounced for the models considered in this study because of the enforced symmetry of the upper and lower skins.) Satisfying the buckling constraints requires a large amount of ± 45 deg material that is not required for strength constraints, as can be seen by comparing the results in Fig. 3 for the constraint cases labeled S + M and S + M + B.

Besides the basic stiffness-to-mass and strength-to-mass ratio advantages of composites over aluminum, composites have another important design advantage over metals that is demonstrated by the results in Table 2. This latter advantage results from the composite material's additional design freedom of changing the relative percentages of the different lamina orientations in a laminate rather than just the total laminate thickness. An example of the advantage of this additional design freedom is seen by comparing the strength, minimum-gage, and buckling constraint design with the strength, minimum-gage, displacement and buckling constraint design for the graphite-epoxy wing in Table 2. The addition of the displacement constraint reduces the displacement at the wing tip by 27% without increasing the mass. By comparing the results of the constraint cases labeled S + M + B and S + M + D + B in Fig. 3, it can be seen that this displacement reduction is accomplished in part by replacing some of the 90 deg lamina material by 0 deg material.

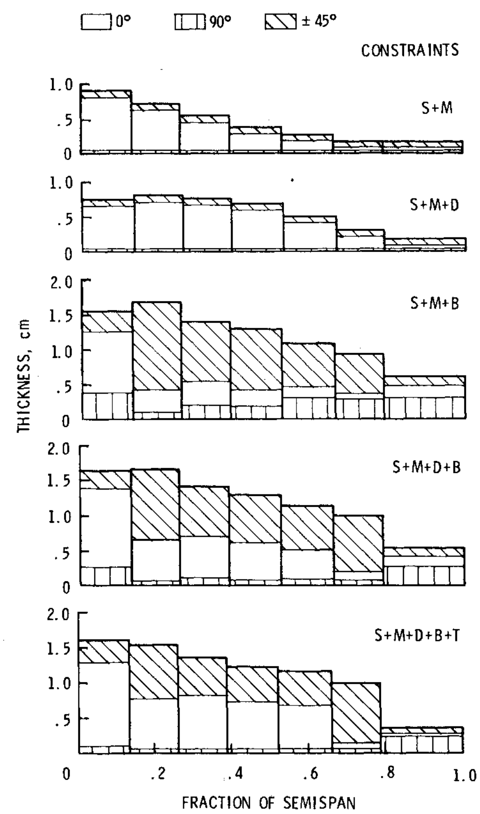

Fig. 3 Effect of constraints on graphite-epoxy, minimum-mass cover thickness distribution. Basic design variable distribution. S = strength, M = minimum gage, D = displacement, B = buckling, and T = twist.

The wing tip twist angles of the designs subjected to strength, minimum-gage, displacement, and buckling constraints are 4.7, 4.6, and 4.0 deg for the graphite-epoxy, graphite-epoxy with boron-epoxy spar caps, and aluminum models, respectively. The reduction in wing tip twist angle required to satisfy the 3 deg wing tip twist constraint is somewhat smaller for the aluminum designs than for the composite designs. However, the addition of twist constraints to the other constraints causes little increase in the masses of the composite designs, but these constraints cause up to a 26% increase in mass for the aluminum design. In fact, the twist constraint is the only critical constraint for the aluminum design. The thickness distribution that satisfies all the constraints for the all-graphite-epoxy model is shown in Fig. 3 by the constraint case labeled S + M + D + B + T. Again, the additional design freedom of composites over metals is demonstrated, since the additional twist constraint is, for the most part, accommodated by changing the relative distribution of the various laminae in the wing rather than increasing the laminate thicknesses.

Effect of Additional Constraints

The fact that this additional design freedom of composites over metals results from the possibility of having different combinations of lamina thicknesses to satisfy design requirements is demonstrated by imposing some additional arbitrary constraints on the all-graphite-epoxy model. First, the design variables controlling the 90 deg laminae of the outer four segments were constrained to have maximum-thickness values of 0.17 cm, compared to values of up to 0.34 cm for the original final design values (see Fig. 3). Then, the stacking sequence of this model was changed by switching the order of the 0 and 90 deg materials in the laminates. The resulting masses and wing tip displacements for these cases and the corresponding case from Table 2 are given in Table 3. The final mass results with these arbitrary constraints are within 1% of the original final results given in Table 2. These additional arbitrary constraints are accommodated with very little mass change, even for the case with all five types of original constraints imposed on the wing. These results suggest that additional constraints, such as those imposed by manufacturing considerations which are similar in nature to the arbitrary constraints, can be accommodated without significant mass increases.

Table 3 Basic graphite-epoxy configuration special cases

Case	Constraints[a] S+M+B	S+M+D+B	S+M+D+B+T
Maximum thickness imposed on 90-deg plies			
Mass, kg	1617	1630	1731
Displacement, cm	206	165	119
New stacking sequence			
Mass, kg	1629	1636	1728
Displacement, cm	214	164	119
Original results from Table 2			
Mass, kg	1612	1624	1722
Displacement, cm	225	164	120

[a]S = strength, M = minimum gage, D = displacement, B = buckling, and T = twist.

Table 4 Refined design variable distribution final design results

Material system	Constraints[a] S+M	S+M+D	S+M+B	S+M+D+B	S+M+D+B+T
Aluminum					
Mass, kg	883	2173	2491	2588	3114
Displacement, cm	498	165	173	165	131
Graphite-epoxy					
Mass, kg	605	840	1329	1383	1481
Displacement, cm	287	165	214	163	131
Graphite-epoxy with boron-epoxy spar caps					
Mass, kg	551	737	1322	1368	1474
Displacement, cm	285	163	231	163	132

[a]S = strength, M = minimum gage, D = displacement, B = buckling, and T = twist.

Graphite-Epoxy vs Boron-Epoxy Spar Caps

There is no significant difference between the final masses of the all-graphite-epoxy wing and the wing with boron-epoxy spar caps when all of the constraints are satisfied. However, the boron-epoxy designs have larger spar caps than the all-graphite-epoxy designs, and the strains in the boron-epoxy spar caps are much lower than the allowable value in all cases. For example, the maximum ratio of strain to allowable strain in the graphite-epoxy spar caps of a typical model subjected to material strength and minimum-gage constraints is 0.998, while it is only 0.651 for the corresponding boron-epoxy model. This difference in the ratios of the strains to the allowable strains indicates that the graphite-epoxy spar caps are sized to satisfy the strain allowable in the spar caps, but the boron-epoxy spar caps are sized to relieve strains in the skin. Such a design cannot be obtained using a fully stressed design technique.

Refined Design Variable Distribution

Local constraints, such as stress or buckling constraints, can be affected by changing the design locally or by global changes that result in internal load redistribution. Such load redistribution may be achieved by chordwise variation of panel and spar stiffnesses. A refined design variable distribution permitting chordwise variation of panel properties is used in this study to determine the effects of load redistribution on the minimum-mass results. This refined design variable distribution also permits spanwise tapering of the spars. The final wing mass and wing tip displacement results for the various constraint combinations are presented in Table 4 for the refined design variable distribution. The spanwise cover thickness distribution along the rear spar for the all-graphite-epoxy model subject to strain, minimum-gage, displacement, panel buckling, and twist constraints is shown in Fig. 4, and the chordwise cover thickness distribution for segments 2, 4, and 6 of Fig. 2 is shown in Fig. 5. Comparing the results in Tables 2 and 4 shows that when all the constraints are imposed, the aluminum final design mass is reduced by only 2%, while the graphite-epoxy final design mass is reduced by 14% as a result of the chordwise cover thickness variation and the spanwise variation. The effect of the load redistribution on the design of the wing panels is studied by isolating one single wing panel as discussed in the next section.

Table 5 Comparison of wing panel and single panel thicknesses

Component, cm	Wing panel	Single panel
90 deg lamina	0.051	0.056
0 deg lamina	1.798	0.826
±45 deg laminae	0.371	1.178
Laminate	2.220	2.060

Single Panel Design

To study the effect of load redistribution on panel designs, a buckling critical cover panel of the all-graphite-epoxy wing subjected to strength, minimum-gage, and buckling constraints is extracted from the wing and sized independent of the wing as a single panel. This single panel is loaded by the in-plane loads carried by the corresponding wing panel for the final minimum-mass wing design. The minimum-mass designs for the single panel and the corresponding wing panel are presented in Table 5, and the single panel is about 7% thinner than the corresponding wing panel. As a further check, an analysis of the wing final design was performed with the single panel design replacing the corresponding wing panel. This analysis shows a significant internal load redistribution in the region of the wing containing the replacement panel. As a result, a number of skin panels violate the buckling constraints; some skin panels, spar caps, and shear webs violate the strength constraints; and the wing tip displacement is increased about 5%. These results indicate the advantage of dealing with stress and local panel buckling constraints

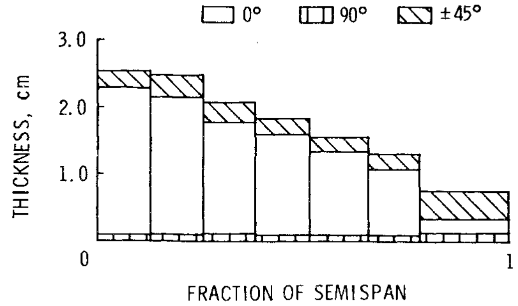

Fig. 4 Spanwise cover skin thickness distribution for the graphite-epoxy, minimum-mass design. Refined design variable distribution.

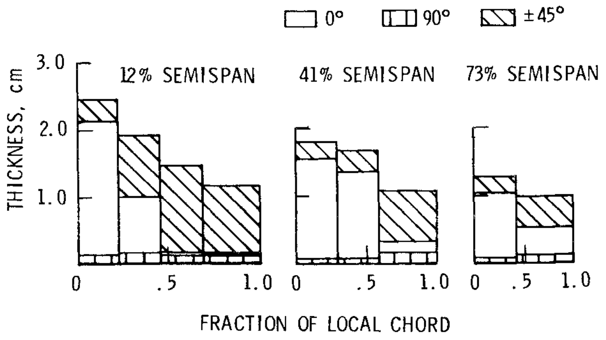

Fig. 5 Chordwise cover skin thickness distribution for the graphite-epoxy, minimum-mass design. Refined design variable distribution.

directly on the global design level rather than using an iterative or sequential design approach, even when no global constraints such as displacement constraints are imposed on the design.

Computational Resources

For the basic design variable distribution, the central processor execution times and central memory storage requirements for this study ranged from 63 s and 132,000 octal locations for the aluminum model with strength and minimum-gage constraints to 299 s and 150,000 octal locations for the graphite-epoxy model with strength, minimum-gage, displacement, buckling, and twist constraints. For the refined design variable distribution, the execution times and core storage requirements ranged from 143 s and 210,000 octal locations for the aluminum models up to 1478 s and 324,000 octal locations for the composite models. All results were obtained with a CDC CYBER 175 digital computer. The number of analyses conducted by the optimization procedure to obtain minimum-mass designs for the constraint cases considered in this study ranged from 17-39 for the basic design variable distribution and from 27-47 for the refined design variable distribution.

Concluding Remarks

An unstiffened panel buckling constraint for balanced, symmetric, composite laminates has been included on the global design level in a computer program for designing wing structures. The program is based on a mathematical programming procedure and also includes material strength, minimum-gage, displacement, and wing twist design constraints. Analytical derivatives of the constraints and constraint approximations are used by the optimization procedure for computational efficiency. For some constraints, such as buckling constraints, the use of constraint approximations may result in significant errors. It was found that it is important to use a conservative constraint approximation minimize the effect of such errors for the optimizatic procedure described herein. A criterion is presented th identifies the more conservative of two forms of constrai approximation that are commonly used.

The optimization procedure is applied to the design ‹ multispar high aspect ratio wing models (with up to 420 fini elements and 100 design variables) made of all graphit epoxy, graphite-epoxy with boron-epoxy spar caps, and a aluminum. Composite material designs have two advantag over aluminum designs: one is the basic advantage of mo favorable stiffness-to-mass and strength-to-mass ratios; tl other is the additional design freedom of selecting the relati percentages of different material orientations in a laminat This design flexibility generally leads to smaller mass ii creases for composite designs than for aluminum desigı when a large number of diverse design constraints are in posed.

Including local constraints, such as stress and pan buckling constraints, on the global design level provides tł advantage of properly accounting for accurate internal loa redistribution during the design process. The importance ‹ accounting for such internal load redistribution is demoı strated by the lighter, but infeasible, design obtained for a independent, single panel designed to satisfy the same coı straints and loads as a corresponding panel in a minimun mass wing design. This advantage is further demonstrated b a minimum-mass wing design where boron-epoxy spar caı are used in an otherwise all graphite-epoxy wing model. The: boron-epoxy spar caps are larger than required to carry tł loads in the spar caps in order to relieve the strains in oth‹ structural components of the wing and to provide an overa weight savings.

Acknowledgment

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